

## Exercise Sheet 4

**Exercise 1.** (for credit, due on 12 October)

Let  $\mu_n = \{\zeta \in \mathbb{C} : \zeta^n = 1\}$  act on  $\mathbb{CP}^1$  by  $z \mapsto \zeta z$  (with  $\infty$  fixed). Let  $\pi : \mathbb{CP}^1 \rightarrow \mathbb{CP}^1/\mu_n$  be the quotient map.

- (1) (2 points) Show that  $\mathbb{CP}^1/\mu_n$  and  $\mathbb{CP}^1$  are biholomorphic.
- (2) (1 point) Compute the degree of  $\pi$ .
- (3) (1 point) Find the ramification points of  $\pi$  and their ramification indices.
- (4) (1 point) Find the branch points of  $\pi$ .

**Exercise 2.** Let  $P(x, y) = x^2 - y^3$  and consider the smooth affine curve

$$C_P := \{(x, y) \in \mathbb{C}^2 : P(x, y) = 0, (\partial_x P, \partial_y P) \neq (0, 0)\}.$$

Show that

$$\nu : \mathbb{C} \setminus \{0\} \rightarrow C_P, \quad \nu(t) = (t^3, t^2)$$

is a biholomorphism.

**Exercise 3.** Fix distinct  $a_1, \dots, a_{2g+2} \in \mathbb{C}$  and let

$$P(x, y) = y^2 - \prod_{j=1}^{2g+2} (x - a_j).$$

Consider the hyperelliptic curve of genus  $g$

$$C_P := \{(x, y) \in \mathbb{C}^2 : P(x, y) = 0, (\partial_x P, \partial_y P) \neq (0, 0)\}.$$

Let  $\pi : C_P \rightarrow \mathbb{C}$  be the projection  $(x, y) \mapsto x$ . Find the ramification points of  $\pi$ , their multiplicities and compute the degree of  $\pi$ .

**Exercise 4.** Can you find a nonconstant polynomial  $P(x, y) \in \mathbb{C}[x, y]$  such that the corresponding affine algebraic curve  $C_P \subset \mathbb{C}^2$  is compact in the Euclidean topology?